Acoustic landmine detection: a 3D poroelastic model

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ABSTRACT

Acoustic waves can be a viable tool for the detection and identification of land mines, unexploded ordnance (UXO) and other buried objects. Design of acoustic instruments and interpretation and processing of acoustic measurements call for accurate numerical models to simulate acoustic wave propagation in a heterogeneous soil with buried objects. Compared with the traditional seismic exploration, high attenuation is unfortunately ubiquitous for shallow surface acoustic measurements because of the loose soil and the fluid in its pore space. To adequately model such acoustic attenuation, we propose a comprehensive multidimensional finite-difference time-domain model to simulate the acoustic wave interactions with land mines and soils based on the Biot theory for poroelastic media. For the truncation of the computational domain, we use the perfectly matched layer (PML). The method is validated by comparison with analytical solutions. Unlike the pure elastic wave model, this efficient PML-FDTD model for poroelastic media incorporates the interactions of waves and the fluid-saturated pore space. Several typical land mine detection measurements are simulated to illustrate the application.

Keywords: poroelastic, land mine detection, staggered-grid, elastic wave, PML, finite-difference

1. INTRODUCTION

Recently acoustic methods have received considerable attention in the characterization of shallow objects. It has been shown that acoustic waves can be a viable tool for the detection and identification of land mines and unexploded ordnance (UXO),¹⁻² (also see some experimental papers in this special issue). The critical issue of acoustic wave coupling into the soil has been addressed by using low-frequency loud-speakers, laser, off-set mechanical source, and electric spark sources.^{2,7} These recent experimental efforts have shown promising results for acoustic landmine detection.

Design of acoustic instruments, interpretation and processing of acoustic measurements call for accurate numerical models to simulate acoustic wave propagation in a heterogeneous soil with buried objects. Conventional methods for acoustic modeling include the popular finite-difference time-domain (FDTD) solution of pure elastic media,³⁻⁴ Unfortunately, the pure elastic model cannot adequately incorporate the attenuation mechanism in its governing equations, although some approximate models are possible.⁵

Compared with the traditional seismic exploration, high attenuation is unfortunately ubiquitous for shallow surface acoustic measurements because of the loose soil and the fluid in its pore space. To adequately model such acoustic attenuation, we propose a comprehensive model to simulate the acoustic wave interactions with land mines and soils based on the Biot theory⁶⁻⁸ for poroelastic media. The finite-difference time-domain method is then used to solve the Biot equations.⁸ Similar to the velocity-stress FD method,⁹ a velocity-strain, finite-difference method is developed in a staggered grid for heterogeneous poroelastic media. In this method, Biot equations⁸ are reformulated into first-order equations to arrive at a leap-frog system in a staggered grid both in time and space domains.

In order to simulate an unbounded medium, an absorbing boundary condition (ABC) must be implemented to truncate the computational domain in numerical algorithms. Several ABCs have been developed for numerical simulation of wave propagation. Cerjan *et al.*¹⁰ introduce a simple damping taper to the boundaries that attenuates outgoing waves. Since this lossy layer is not perfectly matched to the interior region, however, it requires a substantial number of layers for the taper to be effective. Clayton and Engquist¹¹ (CE) use the paraxial approximation to the wave equation to make the boundary transparent to outgoing waves. The CE ABC can lead to instability when

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the Poisson's ratio is greater than two.¹² Since Berenger¹³ proposed the highly effective perfectly matched layer (PML) as an absorbing boundary condition for electromagnetic waves, it has been widely used for finite-difference and finite-element methods. Chew and Liu¹⁴ first proposed the PML for elastic waves in solids, and proved the zero reflections from PML to the regular elastic medium. Hastings *et al.*¹⁵ have independently implemented the PML ABC for two-dimensional problems by using potentials. The PML has also been extended to model acoustic waves and electromagnetic waves in lossy media¹⁶ and to cylindrical and spherical coordinates.¹⁷ Recently, PML has been applied to the second-order Biot's equations for 2-D fluid-saturated poroelastic media,¹⁸ which requires a complicated convolution. In this paper, we apply the PML to 3-D using the first-order PDE system.

The difference between elastic model and poroelastic model is investigated by studying surface wave amplitude variation with offset (AVO) in three different types of soil: dry sand, fully water saturated sand and partly water saturated sand. The interaction of elastic wave with a plastic mine buried in dry sand is simulated. We also simulate two typical configurations for land mine detection, namely the excitation in soil,¹ and the excitation in air.² The results show that the wave responses are significantly affected by the existence of a mine-like object through surface waves. After processing, the target can be detected by using surface acoustic measurements. The 2-D and 3-D poroelastic codes provide a useful tool for design and detection issues in acoustic characterization of buried objects.

2. FORMULATION

2.1. The governing equations

The propagation of acoustic waves in fluid-saturated porous media is different from that in single phase elastic media. In addition to the regular P (compressional) waves and S (shear) waves in solid elastic media, a slow P wave resulting from the relative motion between the solid frame and pore fluid may be present in porous media. Thus energy dissipation in porous media is different from that in solid elastic media. Based on continuum mechanics and macroscopic constitutive relationship, Biot⁶⁻⁸ developed a theory of wave motion in a poroelastic solid saturated with a viscous compressible fluid. Biot's theory was confirmed by Burridge and Keller¹⁹ based on the dynamic equations governing the behavior of medium on a microscopic scale. Plona²⁰ also confirmed Biot's theory through experiments.

For a three-dimensional isotropic, heterogeneous and porous elastic medium, wave propagation is governed by Biot's equations⁸:

$$2 \sum_j \frac{\partial}{\partial x_j} (\mu e_{ij}) + \frac{\partial}{\partial x_i} (\lambda_c e - \alpha M \xi) = \frac{\partial^2}{\partial t^2} (\rho u_i + \rho_f w_i) \quad (1)$$

$$\frac{\partial}{\partial x_i} (\alpha M e - M \xi) = \frac{\partial^2}{\partial t^2} (\rho_f u_i + m w_i) + \frac{\eta}{\kappa} \frac{\partial w_i}{\partial t} \quad (2)$$

where $m = a\rho_f/\phi$ and

$$M = \frac{1}{\frac{\phi}{K_f} + \frac{(\alpha - \phi)}{K_s}}$$

$$\alpha = 1 - \frac{K_b}{K_s}$$

Let $\mathbf{v}^s = \frac{\partial \mathbf{u}}{\partial t}$ be the velocity of the solid particle, $\mathbf{v}^f = \frac{\partial \mathbf{w}}{\partial t}$ be the velocity of the pore fluid relative to the solid frame. Then the second-order equations (1) and (2) can rearranged as the first-order equations,

$$(m\rho - \rho_f^2) \frac{\partial v_i^s}{\partial t} = 2m \sum_j \frac{\partial}{\partial x_j} (\mu e_{ij}) + m \frac{\partial}{\partial x_i} (\lambda_c e - \alpha M \xi) - \rho_f \frac{\partial}{\partial x_i} (\alpha M e - M \xi) + \rho_f \frac{\eta}{\kappa} v_i^f \quad (3)$$

$$(m\rho - \rho_f^2) \frac{\partial v_i^f}{\partial t} = \rho \frac{\partial}{\partial x_i} (\alpha M e - M \xi) - \rho \frac{\eta}{\kappa} v_i^f - 2\rho_f \sum_j \frac{\partial}{\partial x_j} (\mu e_{ij}) - \rho_f \frac{\partial}{\partial x_i} (\lambda_c e - \alpha M \xi) \quad (4)$$

The time derivatives of strains e_{ij} and ξ can be expressed as

$$\frac{\partial e_{ij}}{\partial t} = [(\partial v_j^s / \partial x_i + \partial v_i^s / \partial x_j) / 2] \quad (5)$$

$$\frac{\partial \xi}{\partial t} = -\nabla \cdot \mathbf{v}^f \quad (6)$$

In this work, an explicit second-order finite-difference scheme is applied to convert equations (3) to (6) into a leap-frog system for the strain field e_{ij} , ξ and velocity field $\mathbf{v}^s$ and $\mathbf{v}^f$. With proper absorbing boundary conditions, these equations can be solved numerically for the wave field in an unbounded medium.

Equations (3) to (6) predict the existence of three different waves in fluid-saturated poroelastic media: a shear wave and two compressional waves with a faster and a slower propagation velocities^{6,8}.

2.2. Equations for the PML absorbing boundary condition

In this paper, the perfectly matched layer (PML) is used to truncate the unbounded medium to absorb all outgoing waves. This artificial absorptive medium is introduced in the regular medium by modifying equations (3) to (6) with complex coordinates.^{14,17} In the frequency domain with $e^{-i\omega t}$ convention, a complex coordinate variable is chosen as

$$\tilde{x}_j = \int_0^{x_j} e_j(x_j') dx_j' \quad (7)$$

$$e_j = a_j + i \frac{\omega_j}{\omega} \quad (j = 1, 2, 3) \quad (8)$$

where $a_j \geq 1$ is a scaling factor, and $\omega_{x_j} \geq 0$ is an attenuation factor. The operator $\frac{\partial}{\partial x_j}$ can be expressed in terms of the regular coordinate,

$$\frac{\partial}{\partial \tilde{x}_j} = \frac{1}{e_j} \frac{\partial}{\partial x_j} \quad (9)$$

The PML formulation is to replace x_j in (4)-(6) by the corresponding complex coordinate $\tilde{x}_j$. In a regular non-PML region, $a_j = 1$ and $\omega_j = 0$. In order to simplify PML equations, the field variables are split as the following,

$$v_j^s = \sum_{k=1}^3 v_j^{s(k)}$$

$$v_j^f = \sum_{k=1}^3 v_j^{f(k)}$$

where $v_i^{s(k)}$ and $v_i^{f(k)}$ represent the split field variables containing space derivative $\frac{\partial}{\partial x_k}$ only.

The Biot's equations yield the time-domain PML equations (for component v_1),

$$\begin{aligned} (m\rho - \rho_f^2) \left(\frac{\partial v_1^{s(1)}}{\partial t} + \omega_1 v_1^{s(1)} \right) &= 2m \frac{\partial}{\partial x_1} (\mu e_{11}) + m \frac{\partial}{\partial x_1} (\lambda_c e - \alpha M \xi) \\ &\quad - \rho_f \frac{\partial}{\partial x_1} (\alpha M e - M \xi) \end{aligned} \quad (10)$$

$$(m\rho - \rho_f^2) \left(\frac{\partial v_1^{s(2)}}{\partial t} + \omega_2 v_1^{s(2)} \right) = 2m \frac{\partial}{\partial x_2} (\mu e_{12}) + \rho_f \frac{\eta}{\kappa} (v_1^f + \omega_2 \int_{-\infty}^t v_1^f d\tau) \quad (11)$$

$$(m\rho - \rho_f^2) \left(\frac{\partial v_1^{s(3)}}{\partial t} + \omega_3 v_1^{s(3)} \right) = 2m \frac{\partial}{\partial x_3} (\mu e_{13}) \quad (12)$$

$$\frac{\partial e_{11}}{\partial t} = \frac{\partial v_1}{\partial x_1} - \omega_1 e_{11} \quad (13)$$

$$\frac{\partial e_{12}^{(1)}}{\partial t} = \frac{1}{2} \frac{\partial v_2^s}{\partial x_1} - \omega_1 e_{12}^{(1)} \quad (14)$$

$$\frac{\partial e_{12}^{(2)}}{\partial t} = \frac{1}{2} \frac{\partial v_1^s}{\partial x_2} - \omega_2 e_{12}^{(2)} \quad (15)$$

$$\frac{\partial \xi^{(1)}}{\partial t} = -\frac{\partial v_1^f}{\partial x_1} - \omega_1 \xi^{(1)} \quad (16)$$

$$\frac{\partial \xi^{(2)}}{\partial t} = -\frac{\partial v_2^f}{\partial x_2} - \omega_2 \xi^{(2)} \quad (17)$$

$$\frac{\partial \xi^{(3)}}{\partial t} = -\frac{\partial v_3^f}{\partial x_3} - \omega_3 \xi^{(3)} \quad (18)$$

Similar equations can be obtained in the same way for other components v_2^s , v_3^s , v_1^f , v_2^f , v_3^f , and e_{13} , e_{23} . Within PML region, equations (3) and (4) are split into 18 equations for 3D. Equation (5) is split into 9 equations and equation (6) is split into 3 equations. So the total number of the equations is 30 for the PML region, compared to 13 for the regular interior region. Thus, the memory requirement within the PML region is about two and half times that required by a regular medium for three-dimensional problems. This extra memory requirement in PML region is offset by the effectiveness of PML in absorbing the outgoing waves.

2.3. Finite difference implementation

The governing equations for the PML absorbing boundary condition, such as equations (10)-(18), are first-order partial differential equations for particle velocity and strain. They can be solved with different numerical methods. For the evaluation of seismic and acoustic responses, the finite-difference method is widely used because of its flexibility and simplicity. Here, we use the explicit second-order finite-difference method with a staggered grid in both spatial and temporal domains.

To implement a 3-D finite-difference method to the PML equations, the material parameters and unknown field components are discretized on a regular 3-D grid at the intervals Δx_1 , Δx_2 and Δx_3 . The time domain is also discretized with time step Δt . For the FD implementation of Biot's equations on a staggered grid in Fig. 1, the velocity field components are located at the cell's face centers, while material parameters and normal strains are located at the center of the cell and shear strains are located at the six edge centers. The strain field is computed at $n\Delta t$ and velocity field is computed at $(n + \frac{1}{2})\Delta t$. This staggered grid is similar to that for elastic waves in a solid.^{4,22}

With this discretization, a leap-frog time-stepping system can be obtained. In order to simplify the layout of the formulas, the governing equations with PML boundary condition can be generalized as the first-order differential equation. For examples, (11), (13) and (16) can be rewritten as

$$\frac{\partial v_1^{s(2)}}{\partial t} + c_0 v_1^{s(2)} = c_1 \int_{-\infty}^t v_1^f d\tau + c_2 [2m \frac{\partial}{\partial x_2} (\mu e_{12}) + \rho_f \frac{\eta}{\kappa} (v_1^f)] \quad (19)$$

$$\frac{\partial e_{11}}{\partial t} + \omega_1 e_{11} = \frac{\partial v_2^s}{\partial x_1} \quad (20)$$

$$\frac{\partial \xi^1}{\partial t} + \omega_1 \xi^1 = -\frac{\partial v_1^f}{\partial x_1} \quad (21)$$

where c_0 , c_1 and c_2 space-dependent coefficients. Then the time-stepping equations can be written as,

$$v_1^{s(2)}[j_1, j_2, j_3, (n + \frac{1}{2})] = f_1 v_1^{s(2)}[j_1, j_2, j_3, (n - \frac{1}{2})] + f_2 R_1 \quad (22)$$

$$e_{11}[j_1, j_2, j_3, (n + 1)] = g_1 e_{11}[j_1, j_2, j_3, n] + g_2 R_2 \quad (23)$$

$$\xi^1[j_1, j_2, j_3, (n + 1)] = g_1 \xi^1[j_1, j_2, j_3, n] - g_2 R_3 \quad (24)$$

$$f_1 = \frac{c_0/2 - 1/\Delta t}{1/\Delta t + c_0/2}$$

$$f_2 = \frac{1}{1/\Delta t + c_0/2}$$

$$g_1 = \frac{\omega_1/2 - 1/\Delta t}{1/\Delta t + \omega_1/2}$$

$$g_2 = \frac{1}{1/\Delta t + \omega_1/2}$$

where R_1 , R_2 and R_3 are right-hand sides of equations (19),(20) and (21) respectively. It should be noted that the material parameters in the above equations must be properly averaged in order to arrive at a higher accuracy.⁴ In order to save computer storage, the computational domain is divided into a PML region and an interior region. The absorption of outgoing waves is achieved by the PML region, which consists of several cells of PML materials with a quadratically tapered ω_j profile to increase the attenuation toward the outer boundary. In this paper, ω_j of the PML region is chosen as

$$\omega_j(j) = \frac{(M - 1/2 - j)^2}{(M - 1/2)^2} \omega_{j,max} \quad (25)$$

where $\omega_{j,max}$ is the value at the center of the cell at outermost boundary. At the outer boundary, the velocity components are forced to be zero. For convenience, $\omega_{j,max}$ can be expressed in term of dominant frequency and normalized coefficient. Then equation (25) becomes

$$\omega_j = \begin{cases} 2\pi a_0 f_0 (l_{x_j}/L_{PML})^2, & \text{inside PML,} \\ 0, & \text{outside PML} \end{cases} \quad (26)$$

where f_0 is the dominant frequency of the source, and L_{PML} is the thickness of the PML region, and l_{x_j} is the distance from the interface between the interior region and PML region.

3. NUMERICAL RESULTS

In the following numerical simulations, the source energy is partitioned linearly between the solid and the fluid phases with factors

$$W_f = \phi$$

$$W_s = (1 - \phi)$$

$$W_r = \phi|W_f - W_s|$$

where W_f is the weighting factor for the fluid motion, W_s is for the solid motion and W_r is for the relative motion between solid frame and pore fluid.

3.1. A. Validation of Numerical Results

An analytical solution for the particle velocity field in a homogeneous, fluid-saturated poroelastic medium subject to a point source in 3-D space or a line source in 2-D space can be obtained in a closed form via potential functions.⁹ In a poroelastic medium with an ideal non-viscous fluid, a purely dilatational source will only excite P waves given by scalar potentials,

$$\psi_s(r, t) = \frac{\alpha s(t - \frac{r}{V_f}) + \beta s(t - \frac{r}{V_s})}{4\pi r}$$

$$\psi_f(r, t) = \frac{\alpha A_f s(t - \frac{r}{V_f}) + \beta A_s s(t - \frac{r}{V_s})}{4\pi r}$$

where r is the distance and $s(t)$ is the source time function, A_f and A_s represent the ratios between the solid and fluid motion for the fast P-wave and the slow P-wave, and the coefficients α and β are determined by the regularity conditions. V_f and V_s are the velocities of the fast P-wave and the slow P-wave, respectively. The displacement field in the solid and fluid is then written respectively as $\mathbf{u} = \nabla \psi_s$ and $\mathbf{U} = \nabla \psi_f$.

The validation of the numerical method can be done by comparing the numerical results with the above analytical solution. A homogeneous model of sand saturated with water is considered. A P-wave point source of the first derivative Gaussian time function with $f_0 = 40\text{Hz}$ is located at (0,0,0). Then the solution at (30 m, 30 m, 5 m) is calculated. The numerical result and analytical solution in Fig. 2 have an excellent agreement.

3.2. B. Comparison Between Elastic and Poroelastic Models

In the conventional elastic model, a single-phase medium is considered. There is no energy dissipation in such an elastic medium. In poroelastic model, however, a multi-phase medium (usually solid frame and pore fluid) is considered. The in-phase motion between solid and fluid leads to the regular P wave and S wave, and the out-phase motion between solid and fluid leads to the slow P wave. Because the fluid is viscous, there is energy dissipation between the solid frame and the pore fluid. In Biot's theory, the ratio of viscosity and permeability ($b = \eta/\kappa$) determines this dissipation, as shown in equations (3) and (4). The difference between an elastic model and a poroelastic model is investigated here by studying the effect of b on AVO response in three different ground media: dry sand, fully water saturated sand and partly water saturated sand (70% air and 30% water). For a half-space problem, since the source and receivers are usually on the free surface and the surface wave is the dominant signal at low frequencies, only surface wave is investigated here. In this case, we model the source as a vertical dipole right on the ground surface and displaced horizontally away from the buried object, a scenario similar to the source in.¹

Fig. 3, Fig. 4 and Fig. 5 show the effects of b on the surface wave AVO for dry sand, fully water saturated sand and partly water saturated sand models, respectively. For the dry sand, the difference between elastic and poroelastic models is obvious, but b has little effect on AVO. For the fully water saturated sand model, b has a significant effect on AVO. When b is very large, the AVO response is very close to that of elastic model. For partly water saturated sand model, the difference between elastic and poroelastic model is obvious and b also has a significant effect on AVO.

3.3. C. Acoustic Land Mine Detection Systems

This method is used to model a plastic antipersonnel mine that is difficult to detect with the more conventional electromagnetic induction sensors (*i.e.*, metal detectors). The geometry of the model of a buried mine is shown in Fig. 6. The source can be either on the surface or in the air. There are three arrays of receivers on the surface to pick up the signal of vertical velocity.

The parameters for the plastic mine are chosen as $\rho = 1200 \text{ kg/m}^3$, S-wave velocity $v_{sh} = 1100 \text{ m/s}$ and P-wave velocity $v_p = 2700 \text{ m/s}$. The material in the ground may be considered as a two-phase composite material consisting of granular solid and pore fluid. The nature of this composite varies with environment, geographic location, and with depth below the surface of ground. In this model, the soil is chosen as dry sand and the porosity is 0.35. The fast P wave velocity is 250 m/s and S wave velocity is 87 m/s.

3.3.1. (a). Source on the Ground Surface

To model the land-mine detection system in,¹ a half space with a dipole source on the ground surface is simulated. The source time function is the Blackman-Harris window function with the center frequency 450 Hz. Fig. 7 shows the four snapshots of the total field on the free ground surface at 2.3 ms, 4.0 ms, 6.0 ms and 7.8 ms. The primary wave on these snapshots are surface waves. Compared to the surface waves, the P wave is very weak. The obvious surface wave reflections by the mine are shown on the snapshots at 6.0 ms and 7.8 ms. The influence of the buried mine on the right-propagating surface wave can be seen clearly on the snapshots. There is a decrease on amplitude when the surface wave passes over the buried mine. Fig. 8(a) shows the waveforms of receiver array 1. The reflected surface waves are presented on the first 5 traces. Fig. 8(b) and 8(c) show the waveforms of receiver array 2 and 3 respectively. The decrease of the amplitude can be easily seen on the time traces of receiver 6 to 11.

3.3.2. (b). Source in the Air

Another land-mine detection prototype uses airborne acoustic waves to couple into the ground surface to detect minelike targets.² In our simulation, the model consists of two parts: air (pure fluid) and soil (poroelastic medium) with a plastic land mine. A monopole source is in the air 2 cm above the ground surface. The locations of the receivers are the same as those in the previous model. Fig. 9 shows the four snapshots of vertical velocity field on the ground surface at 2.3 ms, 4.0 ms, 6.0 ms and 7.8 ms. We can see that if a minelike target is present below the surface, the ground vibrational velocity will show distinct changes due to reflection and scattering of surface waves. Figs. 10(a), 10(b) and 10(c) show time traces of the scattered field at receiver arrays 1, 2 and 3 respectively. The signature of the buried land mine is clearly present in these figures. It is interesting to note that both direct arrival and the surface waves are scattered by the object into surface waves.

The above results are shown in time domain. However, the prototype in² actually displays the acoustic energy in the frequency domain. This display can also be easily done by our time-domain solution. Displayed in Fig. 11 is

the energy distribution of the scattered field for a monopole source directly above the mine, showing the presence of the buried object.

4. CONCLUSION

A particle velocity-strain, finite-difference method combined with the perfectly matched layer (PML) has been developed for the simulation of acoustic waves propagating in 3-D poroelastic media. The results show the scheme is stable even if the ratio of the fast P wave velocity to shear wave velocity is greater than 2.

The difference between elastic model and poroelastic model and moisture effect are investigated by studying surface wave amplitude variation with offset (AVO) in three different ground media: dry sand, fully water saturated sand and partly water saturated sand. The difference between these models is significant. For air saturated sand, the attenuation factor has little influence on AVO. For fully or partly water saturated sand, the attenuation factor greatly affects the AVO response.

This numerical method has been used to investigate the interaction of acoustic waves with a buried mine-like object. Two land-mine detection prototypes are simulated: a half space model with a dipole source on the free ground surface and a half space model with a monopole source in the air. The results show that the wave responses are substantially changed by the presence of the mine-like object, and surface waves play as a primary mechanism of detection at the low frequencies.

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Figure 1. The relative locations of field components in a unit cell of staggered grid.

Figure 2. Comparison of analytical and numerical solutions for the vertical velocity component in a homogeneous solid.

Figure 3. Comparison between elastic and poroelastic model for dry sand.

Figure 4. Comparison between elastic and poroelastic model for fully water saturated sand.

Figure 5. Comparison between elastic and poroelastic model for partly water saturated sand.

Figure 6. Geometry of a buried land mine model.

Figure 7. Snapshots at (a) 2.3 ms (b) 4.0 ms (c) 6.0 ms and (d) 7.8 ms. A dipole source is on the free surface.

Figure 8. Waveforms of vertical velocity of solid frame at receiver (a) array 1 (b) array 2 (c) array 3; A dipole source is on the free surface.

Figure 9. Snapshots at (a) 2.3 ms (b) 4.0 ms (c) 6.0 ms and (d) 7.8 ms. A monopole source is in the air.

Figure 10. Waveforms of scattered field of solid frame at receiver (a) array 1 (b) array 2 (c) array 3; A monopole source is in the air.

Figure 11. Energy distribution of the scattered field when the source is in the air above the mine.