A poroelastic model for acoustic landmine detection

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ABSTRACT

Acoustic waves can be a viable tool for the detection and identification of land mines and unexploded ordnance (UXO). Design of acoustic instruments and interpretation and processing of acoustic measurements call for accurate numerical models to simulate acoustic wave propagation in a heterogeneous soil with buried objects. Compared with the traditional seismic exploration, high attenuation is unfortunately ubiquitous for shallow surface acoustic measurements because of the loose soil and the fluid in its pore space. To adequately model such acoustic attenuation, we propose a comprehensive model to simulate the acoustic wave interactions with land mines and soils based on the Biot theory for poroelastic media. The finite-difference time-domain method is then used to solve the Biot equations. For the truncation of the computational domain in the FDTD method, we extend the acoustic and elastic perfectly matched layer (PML) to poroelastic media. Numerical experiments show that, with only 10 cells of PML medium, a high attenuation of about 50 dB can be achieved for outgoing waves. The numerical model is validated by comparison with analytical solutions. Unlike the pure elastic wave model, this efficient PML-FDTD model for poroelastic media incorporates the interactions of waves and the fluid-saturated pore space. The difference between elastic model and poroelastic model is investigated by studying surface wave amplitude variation with offset (AVO) in three different ground media: dry sand, fully water saturated sand and partly water saturated sand. The interaction of elastic wave with a plastic mine buried in dry sand is simulated. The results show that the surface wave is significantly influenced by the existence of a mine-like object. The diffraction of the surface wave can serve as an acoustic target signature.

Keywords: poroelastic, land mine detection, staggered-grid, elastic wave, PML, finite-difference

1. INTRODUCTION

Recently acoustic methods have received considerable attention in the characterization of shallow objects. It has been shown that acoustic waves can be a viable tool for the detection and identification of land mines and unexploded ordnance (UXO). Design of acoustic instruments and interpretation and processing of acoustic measurements call for accurate numerical models to simulate acoustic wave propagation in a heterogeneous soil with buried objects. Conventional methods for acoustic modeling include the popular finite difference time-domain (FDTD) solution of pure elastic media. Unfortunately, the pure elastic model cannot adequately incorporate the attenuation mechanism in its governing equations, although some approximate models are possible.

Compared with the traditional seismic exploration, high attenuation is unfortunately ubiquitous for shallow surface acoustic measurements because of the loose soil and the fluid in its pore space. To adequately model such acoustic attenuation, we propose a comprehensive model to simulate the acoustic wave interactions with land mines and soils based on the Biot theory.¹⁻³ for poroelastic media. The finite-difference time-domain method is then used to solve the Biot equations.³ Like velocity-stress FD method,⁴ a velocity-strain, finite-difference method is developed in a staggered grid for heterogeneous poroelastic media. In this method, Biot's equations³ are reformulated into first-order equations to arrive at a leap-frog system in a staggered grid both in time and space domains. Numerical solutions have been validated by analytical solutions.

In order to simulate an unbounded medium, an absorbing boundary condition (ABC) must be implemented to truncate the computational domain in numerical algorithms. There are many kinds of ABCs developed for numerical simulation of wave propagation. Cerjan *et al.*⁵ introduces a simple damping taper to the boundaries that attenuates the outgoing waves. Since this lossy layer is not perfectly matched to the interior region, however, it requires a substantial number of layers for the taper to be effective. Clayton and Engquist⁶ (CE) use the paraxial approximation to the wave equation to make the boundary transparent to outgoing waves. The CE ABC can lead

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to instability when the Poisson's ratio is greater than two.⁷ Since Berenger⁸ proposed the highly effective perfectly matched layer (PML) as an absorbing boundary condition for electromagnetic waves, it has been widely used for finite-difference and finite-element methods. Chew and Liu⁹ first proposed the PML for elastic waves in solids, and proved the zero reflections from PML to the regular elastic medium. Hastings *et al.*¹¹ have independently implemented the PML ABC for two-dimensional problems by using potentials. The PML has also been extended to model acoustic waves and electromagnetic waves in lossy media.¹² PML has been applied to the second-order Biot's equation for fluid-saturated poroelastic media,¹³ which requires a complicated convolution. In this paper, as the PML is developed for the first-order system, incorporation of PML becomes much simpler. The performance of PML is investigated. The effectiveness of this ABC is confirmed by examining the reflection from the boundary.

The difference between elastic model and poroelastic model is investigated by studying surface wave amplitude variation with offset (AVO) in three different ground media: dry sand, fully water saturated sand and partly water saturated sand. The interaction of elastic wave with a plastic mine buried in dry sand is simulated. The results show that the wave responses is significantly influenced by the existence of a mine-like object. After processing, the target can be detected by using surface acoustic measurements.

2. FORMULATION

2.1. The governing equations

The propagation of acoustic waves in porous and fluid-saturated media is different from that in single phase elastic media. In addition to the regular P waves and S waves in solid elastic media, a slow P wave which results from the relative motion between solid frame and fluid may be present in porous media. Thus the pattern of energy dissipation in porous media is different from that in solid elastic media. Based on continuum mechanics and macroscopic constitutive relationship, Biot¹⁻³ developed a theory of wave motion in a porous elastic solid saturated with a viscous compressible fluid. Biot's theory was confirmed by Burridge and Keller¹⁴ based on the dynamic equations which govern the behavior of medium on a microscopic scale. Plona¹⁵ also confirmed Biot's theory through experiments.

In an isotropic, heterogeneous porous elastic medium, the parameters describing the physical properties of the medium are as follows,

μ	shear modulus of dry porous matrix
λ_c	Lame constant of saturated matrix
ϕ	porosity
η	viscosity
κ	permeability
ρ	the overall density of the saturated medium determined by $\rho_f\phi + (1 - \phi)\rho_s$
ρ_s	density of solid material
ρ_f	density of fluid
a	tortuosity of the matrix
K_s	bulk modulus of the solid
K_f	bulk modulus of the fluid
K_b	bulk modulus of the dry porous frame

The macroscopic displacements and strains are defined as

u_i	i th component of displacements of solid particle
U_i	i th component of displacements of fluid particle
w_i	i th component of relative displacement, $w_i = \phi(U_i - u_i)$
e_{ij}	ij component of strain tensor in porous medium, $e_{ij} = [(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j})/2]$
e	$e = \sum_{i=1,2,3} e_{ii}$
ξ	dilatation for the relative motion, $\xi = -\sum_{i=1,2,3} \frac{\partial w_i}{\partial x_i}$

For a three-dimensional isotropic, heterogeneous and porous elastic medium, wave propagation is governed by Biot's equations³:

$$2 \sum_j \frac{\partial}{\partial x_j} (\mu e_{ij}) + \frac{\partial}{\partial x_i} (\lambda_c e - \alpha M \xi) = \frac{\partial^2}{\partial t^2} (\rho u_i + \rho_f w_i) \quad (1)$$

$$\frac{\partial}{\partial x_i}(\alpha M e - M \xi) = \frac{\partial^2}{\partial t^2}(\rho_f u_i + m w_i) + \frac{\eta}{\kappa} \frac{\partial w_i}{\partial t} \quad (2)$$

where $m = a\rho_f/\phi$ and

$$M = \frac{1}{\frac{\phi}{K_f} + \frac{(\alpha-\phi)}{K_s}}$$

$$\alpha = 1 - \frac{K_b}{K_s}$$

Let $\mathbf{v}^s$ be the velocity of the solid particle, $\mathbf{v}^f$ be the velocity of the pore fluid relative to the solid frame. Then the second-order equations (1) and (2) can be rearranged as the first-order equations,

$$\begin{aligned} (m\rho - \rho_f^2) \frac{\partial v_i^s}{\partial t} &= 2m \sum_j \frac{\partial}{\partial x_j}(\mu e_{ij}) + m \frac{\partial}{\partial x_i}(\lambda_c e - \alpha M \xi) \\ &\quad - \rho_f \frac{\partial}{\partial x_i}(\alpha M e - M \xi) + \rho_f \frac{\eta}{\kappa} v_i^f \end{aligned} \quad (3)$$

$$\begin{aligned} (m\rho - \rho_f^2) \frac{\partial v_i^f}{\partial t} &= \rho \frac{\partial}{\partial x_i}(\alpha M e - M \xi) - \rho \frac{\eta}{\kappa} v_i^f \\ &\quad - 2\rho_f \sum_j \frac{\partial}{\partial x_j}(\mu e_{ij}) - \rho_f \frac{\partial}{\partial x_i}(\lambda_c e - \alpha M \xi) \end{aligned} \quad (4)$$

The time derivatives of strains e_{ij} and ξ can be expressed as

$$\frac{\partial e_{ij}}{\partial t} = [(\partial v_j^s / \partial x_i + \partial v_i^s / \partial x_j) / 2] \quad (5)$$

$$\frac{\partial \xi}{\partial t} = -\nabla \cdot \mathbf{v}^f \quad (6)$$

In the explicit first-order finite-difference schemes, equations (3) to (6) consist of a leap-frog system for the strain field e_{ij} , ξ and velocity field $\mathbf{v}^s$ and $\mathbf{v}^f$. With proper absorbing boundary conditions, these equations can be solved numerically for the wave field in an unbounded medium.

Equations (3) to (6) predict the existence of three different waves in fluid-saturated poroelastic media: a shear wave and two compressional waves with a faster and a slower propagation velocities.

2.2. Equations for the PML absorbing boundary condition

In this paper, the perfectly matched layer (PML) will be used to truncate the unbounded medium, absorbing all outgoing waves. The artificial absorptive medium is introduced in the regular medium by modifying equations (3) to (6) with complex coordinates.^{10,11} In the frequency domain, a complex coordinate variable is chosen as

$$\tilde{x}_i = \int_0^{x_i} e_i(x_i') dx_i' \quad (7)$$

$$e_i = a_i + j \frac{\omega_i}{\omega} \quad (i = 1, 2, 3) \quad (8)$$

where $a_i \geq 1$ is a scaling factor, and $\omega_{x_i} \geq 0$ is an attenuation factor. The operator $\frac{\partial}{\partial x_i}$ can be expressed in terms of the regular coordinate,

$$\frac{\partial}{\partial \tilde{x}_i} = \frac{1}{e_i} \frac{\partial}{\partial x_i} \quad (9)$$

The PML formulation is to replace x_i in (4)-(6) by the corresponding complex coordinate $\tilde{x}_i$. In a PML region, the real part a_i is scaling factor, and the imaginary part ω_i represents a loss in the PML. In a regular non-PML region, $a_i = 1$ and $\omega_i = 0$.

In order to make equations with PML simple, the equations and variables are split as the following,

$$v_i^s = \sum_{k=1}^3 v_i^{s(k)}$$

$$v_i^f = \sum_{k=1}^3 v_i^{f(k)}$$

where $v_i^{s(k)}$ and $v_i^{f(k)}$ represent the split field variables containing space derivative $\frac{\partial}{\partial x_k}$ only. For example, equation (3) for $i = 1$ can be split into the following 3 equations,

$$\begin{aligned} (m\rho - \rho_f^2) \frac{\partial v_1^{s(1)}}{\partial t} &= 2m \frac{\partial}{\partial x_1} (\mu e_{11}) + m \frac{\partial}{\partial x_1} (\lambda_c e - \alpha M \xi) - \rho_f \frac{\partial}{\partial x_1} (\alpha M e - M \xi) \\ (m\rho - \rho_f^2) \frac{\partial v_1^{s(2)}}{\partial t} &= 2m \frac{\partial}{\partial x_2} (\mu e_{12}) + \rho_f \frac{\eta}{\kappa} v_1^f \\ (m\rho - \rho_f^2) \frac{\partial v_1^{s(3)}}{\partial t} &= 2m \frac{\partial}{\partial x_3} (\mu e_{13}) \end{aligned}$$

The diagonal strain components e_{ii} need not be split. However, other strain components have to be split as $e_{ij} = \sum_{k=1}^3 e_{ij}^{(k)}$ for $i \neq j$, and $\xi = \sum_{k=1}^3 \xi^{(k)}$, for example,

$$\begin{aligned} \frac{\partial e_{ii}}{\partial t} &= \frac{\partial v_i^s}{\partial x_i} \\ e_{12} &= e_{12}^{(1)} + e_{12}^{(2)} \\ \frac{\partial e_{12}^{(1)}}{\partial t} &= \frac{1}{2} \frac{\partial v_2^s}{\partial x_1} \\ \frac{\partial e_{12}^{(2)}}{\partial t} &= \frac{1}{2} \frac{\partial v_1^s}{\partial x_2} \\ \xi &= \xi^{(1)} + \xi^{(2)} + \xi^{(3)} \\ \frac{\partial \xi^{(1)}}{\partial t} &= -\frac{\partial v_1^f}{\partial x_1} \\ \frac{\partial \xi^{(2)}}{\partial t} &= -\frac{\partial v_2^f}{\partial x_2} \\ \frac{\partial \xi^{(3)}}{\partial t} &= -\frac{\partial v_3^f}{\partial x_3} \end{aligned}$$

In frequency domain PML formulation, after x_i is replaced by $\tilde{x}_i$, equations for v_1^s , for example, can be rewritten as

$$\begin{aligned} (m\rho - \rho_f^2)(-j\omega)(1 + j\frac{\omega_{x_1}}{\omega})v_1^{s(1)} &= 2m \frac{\partial}{\partial x_1} (\mu e_{11}) + m \frac{\partial}{\partial x_1} (\lambda_c e - \alpha M \xi) - \rho_f \frac{\partial}{\partial x_1} (\alpha M e - M \xi) \\ (m\rho - \rho_f^2)(-j\omega)(1 + j\frac{\omega_{x_1}}{\omega})v_1^{s(2)} &= 2m \frac{\partial}{\partial x_2} (\mu e_{12}) + (1 + j\frac{\omega_1}{\omega})\rho_f \frac{\eta}{\kappa} v_1^f \\ (m\rho - \rho_f^2)(-j\omega)(1 + j\frac{\omega_1}{\omega})v_1^{s(3)} &= 2m \frac{\partial}{\partial x_3} (\mu e_{13}) \end{aligned}$$

By taking inverse Fourier transform, the above equations yield the time-domain PML equations,

$$\begin{aligned} (m\rho - \rho_f^2) \left(\frac{\partial v_1^{s(1)}}{\partial t} + \omega_1 v_1^{s(1)} \right) &= 2m \frac{\partial}{\partial x_1} (\mu e_{11}) + m \frac{\partial}{\partial x_1} (\lambda_c e - \alpha M \xi) \\ &\quad - \rho_f \frac{\partial}{\partial x_1} (\alpha M e - M \xi) \end{aligned} \tag{10}$$

$$(m\rho - \rho_f^2)(\frac{\partial v_1^{s(2)}}{\partial t} + \omega_1 v_1^{s(2)}) = 2m \frac{\partial}{\partial x_2}(\mu e_{12}) + \rho_f \frac{\eta}{\kappa}(v_1^f + \omega_2 \int_{-\infty}^t v_1^f d\tau) \quad (11)$$

$$(m\rho - \rho_f^2)(\frac{\partial v_1^{s(3)}}{\partial t} + \omega_1 v_1^{s(3)}) = 2m \frac{\partial}{\partial x_3}(\mu e_{13}) \quad (12)$$

$$\frac{\partial e_{11}}{\partial t} = \frac{\partial v_1}{\partial x_1} - \omega_1 e_{11} \quad (13)$$

$$\frac{\partial e_{12}^{(1)}}{\partial t} = \frac{1}{2} \frac{\partial v_2^s}{\partial x_1} - \omega_1 e_{12}^{(1)} \quad (14)$$

$$\frac{\partial e_{12}^{(2)}}{\partial t} = \frac{1}{2} \frac{\partial v_1^s}{\partial x_2} - \omega_2 e_{12}^{(2)} \quad (15)$$

$$\frac{\partial \xi^{(1)}}{\partial t} = -\frac{\partial v_1^f}{\partial x_1} - \omega_1 \xi^{(1)} \quad (16)$$

$$\frac{\partial \xi^{(2)}}{\partial t} = -\frac{\partial v_2^f}{\partial x_2} - \omega_2 \xi^{(2)} \quad (17)$$

$$\frac{\partial \xi^{(3)}}{\partial t} = -\frac{\partial v_3^f}{\partial x_3} - \omega_3 \xi^{(3)} \quad (18)$$

Similar equations can be obtained in the same way for other components v_2^s , v_3^s , v_1^f , v_2^f , v_3^f , and e_{13} , e_{23} . Within PML region, equations (3) and (4) are split into 18 equations for 3D. Equation (5) is split into 9 equations and equation (6) is split into 3 equations. So the total number of the equations is 30 for the PML region, compared to 13 for the regular interior region. Thus, the memory requirement within the PML region is about two and half times that required by a regular medium for three-dimensional problems. This extra memory requirement in PML region is offset by the effectiveness of PML in absorbing the outgoing waves.

2.3. Finite difference implementation

The governing equations for the PML absorbing boundary condition, such as equations (10)-(18) are first-order partial differential equations for particle velocity and strain. They can be solved with different numerical methods. For the evaluation of seismic and acoustic responses of specific models, the accuracy and convenience of the numerical method are of primary concern. The finite-difference method is widely used in wave modeling because of its flexibility and accuracy. For the first-order, leap-frog system of equations (3) to (6), the explicit finite-difference method is used.

To implement a 3-D finite-difference solution to the equations with PML, the material parameters and unknown field components are discretized on a regular 3-D grid at the intervals Δx_1 , Δx_2 and Δx_3 . The time domain is also discretized with time step Δt . There are two kinds of operators to approximate the first-order derivatives, *i.e.*, the centered grid and staggered grid. Because the centered-grid operator to perform first derivatives are less accurate than staggered grid operators,¹⁶ a staggered grid is used in this paper. For the FD implementation of Biot's equations on a staggered grid in Fig. 1, the velocity field components are located at the cell's face centers, while material parameters and normal strains are located at the center of the cell and shear strains are located the at six edge centers. Strain field is computed at $n\Delta t$ and velocity field is computed at $(n + \frac{1}{2})\Delta t$. This staggered grid is similar to that for elastic waves in a solid.^{17,18}

With this discretization, the leap-frog system can be written in a time-stepping form. In order to make the layout of the formulas simple, the governing equations with PML boundary conditions can be generalized as the first-order differential equation. For examples, (11), (13) and (16) can be rewritten as

$$\frac{\partial v_1^{s(2)}}{\partial t} + c_0 v_1^{s(2)} = c_1 \int_{-\infty}^t v_1^f d\tau + c_2 [2m \frac{\partial}{\partial x_2}(\mu e_{12}) + \rho_f \frac{\eta}{\kappa}(v_1^f)] \quad (19)$$

$$\frac{\partial e_{11}}{\partial t} + \omega_1 e_{11} = \frac{\partial v_2^s}{\partial x_1} \quad (20)$$

$$\frac{\partial \xi^1}{\partial t} + \omega_1 \xi^1 = -\frac{\partial v_1^f}{\partial x_1} \quad (21)$$

where c_0 , c_1 and c_2 space-dependent coefficients. The time-stepping equations can be written as,

$$v_1^{s(2)}[j_1, j_2, j_3, (n + \frac{1}{2})] = f_1 v_1^{s(2)}[j_1, j_2, j_3, (n - \frac{1}{2})] + f_2 R_1 \quad (22)$$

$$e_{11}[j_1, j_2, j_3, (n + 1)] = g_1 e_{11}[j_1, j_2, j_3, n] + g_2 R_2 \quad (23)$$

$$\xi^1[j_1, j_2, j_3, (n + 1)] = g_1 \xi^1[j_1, j_2, j_3, n] - g_2 R_3 \quad (24)$$

$$f_1 = \frac{c_0/2 - 1/\Delta t}{1/\Delta t + c_0/2}$$

$$f_2 = \frac{1}{1/\Delta t + c_0/2}$$

$$g_1 = \frac{\omega_1/2 - 1/\Delta t}{1/\Delta t + \omega_1/2}$$

$$g_2 = \frac{1}{1/\Delta t + \omega_1/2}$$

where R_1 , R_2 and R_3 are right-hand sides of equations (19), (20) and (21) respectively. It should be noted that the material parameters in the above equations must be properly averaged in order to arrive at a higher accuracy.¹⁷ In order to incorporate the PML boundary condition, the computational domain is divided into a PML region and an interior region. The absorption of outgoing waves is achieved by the PML region, which consists of several cells of mathematically defined materials with a quadratically tapered ω_j profile to increase the attenuation toward the outer boundary. In this paper, ω_j of the PML region is chosen as

$$\omega_i(j_i) = \frac{(M - 1/2 - j_i)^2}{(M - 1/2)^2} \omega_{i,max} \quad (25)$$

where $\omega_{i,max}$ is the value at the center of the cell at outermost boundary. At the outer boundary, the velocity components and shear strain are forced to be zero. For convenience, $\omega_{i,max}$ can be expressed in term of dominant frequency and normalized coefficient. Then equation (25) becomes

$$\omega_i = \begin{cases} 2\pi a_0 f_0 (l_{x_i}/L_{PML})^2, & \text{inside PML,} \\ 0, & \text{outside PML} \end{cases} \quad (26)$$

where f_0 is the dominant frequency of the source, and L_{PML} is the thickness of the PML region, and l_{x_i} is the distance from the interface between the interior region and PML region.

3. NUMERICAL RESULTS

The finite-difference algorithm on a staggered grid has a higher accuracy than on a centered grid. Thus, the PML region can be made thinner with a staggered grid with a better absorption than with a centered grid. In this paper, the length of PML region is chosen to be 10 cells. The FD algorithm is illustrated by its two-dimensional implementation.

In the following numerical simulations, the source will be a monopole source for a homogeneous medium and a dipole source for a half space. The source time function is the first derivative of a Gaussian function

$$S(t) = (t - t_0) e^{-\pi^2 f_0^2 (t - t_0)^2}$$

where f_0 is the predetermined dominant frequency, and t_0 the time shift.

The source energy is partitioned linearly between the solid and the fluid phases with factors

$$\begin{aligned} W_f &= \phi \\ W_s &= (1 - \phi) \\ W_r &= \phi |W_f - W_s| \end{aligned}$$

where W_f is the weighting factor for the fluid motion, W_s is for the solid motion and W_r is for the relative motion between solid frame and pore fluid.

3.1. PML performance

The effectiveness of the absorbing boundary condition is an essential factor for the successful numerical simulation. A good absorbing boundary condition has the characteristics of effective absorption of outgoing waves without requiring a large memory. In this paper, the performance of the PML is investigated on a homogeneous, fluid-saturated poroelastic medium by comparing numerical solutions of PML model with reference solutions that do not have reflections. The optimized a_0 can be obtained by examining the attenuation of the wave field in the PML region.

The homogeneous model for PML performance test is a sandstone fully saturated with water and has the properties of $\rho_s = 2650 \text{ kg/m}^3$, $\rho_f = 1040 \text{ kg/m}^3$, and porosity $\phi = 0.3$. This model has the wave properties as $v_{fast} = 2365.7 \text{ m/s}$ for fast P wave, $v_{slow} = 776.95 \text{ m/s}$ for slow P wave and $v_{shear} = 960.5 \text{ m/s}$ for shear wave.

We first study the performance of the PML ABC for a homogeneous medium. Fig. 2 compares a numerical solution at a location 5-cell away from PML interface with a reference solution. (The reference solution is obtained from a much larger model with Dirichlet boundary in which reflections have not arrived within the time window of interest.) No apparent reflections are present in the numerical solution with the PML boundary. Compared to the incident signal, the reflection is about 50dB down. Another advantage of PML absorbing boundary condition is its superior stability. For this particular model, a high ratio of $v_{fast}/v_{shear} = 2.46$ will cause instability problem for CE boundary condition¹⁰.

3.2. Validation of numerical results

An analytical solution for the particle velocity field in a homogeneous, fluid-saturated poroelastic medium subject to a point source in 3-D space or a line source in 2-D space can be derived.⁴ The particle velocity is obtained in a closed form via potential functions.

For Biot's equations, it is convenient to solve for the particle velocity through potential functions. The velocity of particles and body force at source can be expressed in term of potentials as

$$\mathbf{u} = \nabla\psi_s + \nabla \times \Psi_s$$

$$\mathbf{U} = \nabla\psi_f + \nabla \times \Psi_f$$

$$\mathbf{f} = \nabla\Phi + \nabla \times \Psi$$

where $\nabla \cdot \Psi_f = 0$, $\nabla \cdot \Psi_s = 0$ and $\nabla \cdot \Psi = 0$ and describe the rotational potentials, while ψ_s , ψ_f and Φ describe the dilatational potentials. If the source is purely dilatational, then its rotational component disappears. In the time domain, for a purely P-wave point source and ideal non-viscous fluid, the potential can be expressed as

$$\psi_s(r, t) = \frac{\alpha s(t - \frac{r}{V_f}) + \beta s(t - \frac{r}{V_s})}{4\pi r}$$

$$\psi_s(r, t) = \frac{\alpha A_f s(t - \frac{r}{V_f}) + \beta A_s s(t - \frac{r}{V_s})}{4\pi r}$$

where r is the distance and $s(t)$ is the source time function. A_f and A_s represent the ratios between the solid and fluid motion for the fast P-wave and the slow P-wave. The coefficients α and β are determined by the regularity conditions. V_f and V_s are the velocities of the fast P-wave and the slow P-wave, respectively.

In two dimensions, for a pure P-wave line source along y -axis, the solution can be obtained by integrating the point source solution in y direction. In $x - z$ plane, the dilatational potentials are

$$\psi_s(x, z, t) = \frac{\alpha H(t - \frac{r}{V_f})}{2\pi} \int_{r/V_f}^t \frac{s(t - \tau)}{\sqrt{\tau^2 - r^2/V_f^2}} d\tau + \frac{\beta H(t - \frac{r}{V_s})}{2\pi} \int_{r/V_s}^t \frac{s(t - \tau)}{\sqrt{\tau^2 - r^2/V_s^2}} d\tau$$

$$\psi_f(x, z, t) = \frac{\alpha A_f H(t - \frac{r}{V_f})}{2\pi} \int_{r/V_f}^t \frac{s(t - \tau)}{\sqrt{\tau^2 - r^2/V_f^2}} d\tau + \frac{\beta A_s H(t - \frac{r}{V_s})}{2\pi} \int_{r/V_s}^t \frac{s(t - \tau)}{\sqrt{\tau^2 - r^2/V_s^2}} d\tau$$

where $H()$ is the Heaviside step function and $r = \sqrt{x^2 + z^2}$. Once the potential functions are available, the velocity can be easily obtained by taking gradient of potential functions.

The validation of the numerical method can be done by comparing the numerical results with the above analytical solution. A homogeneous model whose parameters are the same as the previous model is considered. A P-wave line source of the first derivative Gaussian time function with $f_0 = 40\text{Hz}$ is located at (0,0). Then the solution at (30 m, 30 m) is calculated. The numerical result and analytical solution in Fig. 3 have an excellent agreement.

3.3. Comparison between elastic and poroelastic models

In the conventional elastic model, a single-phase medium is considered. There is no energy dissipation in such an elastic medium. In poroelastic model, however, a multi-phase medium (usually solid frame and pore fluid) is considered. The in-phase motion between solid and fluid leads to the regular P wave and S wave, and the out-phase motion between solid and fluid leads to the slow P wave. Because the fluid is always viscous, there is energy dissipation between the solid frame and the pore fluid. In Biot's theory, the ratio of viscosity and permibility ($b = \eta/\kappa$) determines this dissipation. The difference between an elastic model and a poroelastic model is investigated by studying the influence of b on AVO response in three different ground media: dry sand, fully water saturated sand and partly water saturated sand (70% air and 30% water). For a half-space problem, since the source and receivers are usually on the free surface and the surface wave is the dominant signal, only surface wave is investigated.

Fig. 4, Fig. 5 and Fig. 6 show the influences of b on the surface wave AVO for dry sand, fully water saturated sand and partly water saturated sand models, respectively. The difference between elastic and poroelastic models for dry sand is obvious, but b has very little influence on AVO. For the fully water saturated sand model, b has a significant effect on AVO. When b is very large, the AVO response is very close to that of elastic model. For partly water saturated sand model, the difference between elastic and poroelastic model is obvious and b also has an obvious effect on AVO.

3.4. Applications

In this section, this method is used to model a plastic antipersonnel mine that is difficult to detect with the more conventional electromagnetic induction sensors (*i.e.*, metal detectors). The geometry of the model of a buried mine is shown in Fig. 7.

The parameters for the plastic mine are chosen as $\rho = 1200 \text{ kg/m}^3$, S-wave velocity $v_{sh} = 1100 \text{ m/s}$ and P-wave velocity $v_p = 2700 \text{ m/s}$. The material in the ground may be considered as a two-phase composite material consisting of granular solid and pore fluid. The nature of this composite varies with environment, geographic location, and with depth below the surface of ground. In this model, the soil is chosen as dry sand, and the fast P wave velocity is 250 m/s and S wave velocity is 87 m/s . The peak frequency of the source is 450 Hz .

Fig. 8 shows the four snapshots at 4.55 ms, 10.4 ms, 11.05 ms and 15.6 ms. The obvious surface wave reflections are shown on the snapshot at 15.6 ms. Fig. 9 shows the seismograms of the vertical velocity of solid frame at the surface. Fig. 10 shows the scattered fields at 4.55 ms, 10.4 ms, 11.05 ms and 15.6 ms. Fig. 11 shows the seismograms of the scattered fields. Compared to the surface wave, the P wave is very weak. It is the surface wave reflection from the mine that is useful for land mine detection at a relatively low frequency.

4. CONCLUSION

A comprehensive model is developed to simulate the acoustic wave interactions with land mines and soils based on the Biot theory for poroelastic media. The velocity-strain, finite-difference method is then used to solve the Biot equations. For the truncation of the computational domain in the FDTD method, we extend the acoustic and elastic perfectly matched layer (PML) to poroelastic media. Numerical experiments show that, with only 10 cells of PML medium, a high attenuation of about 50 dB can be achieved for outgoing waves. The numerical model is validated by comparison with analytical solutions. Unlike the pure elastic wave model, this efficient PML-FDTD model for poroelastic media incorporates the interactions of waves and the fluid-saturated pore space.

The difference between elastic model and poroelastic model is investigated by studying surface wave amplitude variation with offset in three different ground media: dry sand, fully water saturated sand and partly water saturated sand. For dry sand, b has little influence on surface wave AVO. But for wet sand, b has a profound effect on surface AVO. This numerical method has been used to simulate the interaction of elastic waves with a buried mine-like object in dry sand. The results show that the surface wave response is significantly influenced by the existence of a mine-like object. This numerical model provides a useful tool for system design and data interpretation in acoustic demining.

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Figure 1. The relative locations of field components in a unit cell of staggered grid.

Figure 2. Comparison of a numerical result with PML ABC with a reference solution.

Figure 3. Comparison of analytical and numerical solutions for the vertical velocity component in a homogeneous solid.

Figure 4. Comparison between elastic and poroelastic model for dry sand.

Figure 5. Comparison between elastic and poroelastic model for fully water saturated sand.

Figure 6. Comparison between elastic and poroelastic model for partly water saturated sand.

Figure 7. Geometry of a buried land mine model.

Figure 8. Snapshots at (a) 4.55 ms; (b) 10.4 ms; (c) 11.05 ms; (d) 15.6 ms.

Figure 9. Seismograms of vertical velocity of solid frame.

Figure 10. Scattered field at (a) 4.55 ms; (b) 10.4 ms; (c) 11.05 ms; (d) 15.6 ms.

Figure 11. Seismograms of scattered vertical velocity field of solid frame.