A subspace decomposition technique to improve GPR imaging of anti-personnel mines

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ABSTRACT

Ground-reflected clutter is often a performance-limiting factor in ground-penetrating radar (GPR) detection of near-surface targets including anti-personnel mines. When a down-looking antenna is scanned across the surface this reflection produces a strong band in the image, which obscures shallow targets. Imperfections in the system impulse response (e.g., antenna ringing and cable reflections) can produce similar bands. Radar images of buried targets can be degraded by these forms of clutter.

A multi-step process is described to improve imaging of near-surface objects. After range compression of the transmitted pulse, an estimate of the ground-reflected signal is obtained. Slow variations with scan position in the (complex) amplitude and time delay of this reflection are estimated in overlapping piecewise segments using a polynomial model. The resulting reflection estimate is subtracted from the data. Residual clutter is further reduced by using a subspace decomposition technique based on the generalized singular value decomposition (GSVD). In effect, the data are projected onto a space that is orthogonal to the reflection. Both surface reflections and ringing in the system impulse response are removed in this manner. The resulting cleaned data are then used with a conventional imaging algorithm (migration). The procedure is illustrated with data acquired by an OSU-developed GPR at the US Army Fort A.P. Hill, Site 71A.

Keywords: ground penetrating radar, land mines, generalized singular value decomposition, subsurface imaging

1. INTRODUCTION

Radar imaging is often an attractive technique for processing ground penetrating radar (GPR) data. When a downlooking antenna is scanned over a small target, a characteristic hyperbolic arc appears in the time-domain data. Using various algorithms one can refocus the arc energy back to the target location, thereby improving the signal-to-noise ratio and better localizing the target.

Imaging with GPR data is frequently hampered by clutter. This is particularly true for shallow targets. Attempts to detect near-surface buried targets (e.g., anti-personnel mines) using GPR are often limited by the ground reflected wave. When a downward-looking antenna is scanned across the surface this reflection produces a strong band in the time-domain data, which obscures shallow targets. Imperfections in the system impulse response (e.g., antenna ringing and cable reflections) can produce similar bands. Removing these clutter contributions is an essential preprocessing step in obtaining useful data.

In this work we describe a new technique for eliminating ground reflected clutter and system ringing from GPR data. In Section 2 we show that the reflection from uneven ground can be suppressed by subtracting an estimate of the spatially varying return. Once this dominant contribution is removed, residual system clutter can be suppressed using a subspace decomposition technique described in Section 3. In Section 4 we show that images obtained from the clutter-suppressed data are significantly improved. Finally, examples of the clutter reduction and imaging processes are presented in Section 5. The data presented in this work were acquired at the US Army Ft. A.P. Hill (Site 71A) using a GPR developed by the Ohio State University.¹

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2. SUPPRESSING THE GROUND REFLECTION

Several clutter reduction techniques have been discussed in the literature, but none of these methods perform satisfactorily when applied to GPR data collected over shallowly buried mines in nonuniform ground. Perhaps the simplest technique is time gating, a concept that is well known in radar and needs no explanation here. Time gating has been used to remove the ground reflection from GPR measurements of deep targets such as buried pipes, unexploded ordnance and tunnels, but it is inappropriate when the targets are very shallow.

For ensemble average subtraction² one averages the time-domain signatures acquired as the antenna is scanned over (mostly target-free) ground, and subtracts that average from the data. This approach is effective if the ground is flat and uniform over the antenna path - a situation rarely encountered in the field. It also tends to remove parts of the target if the scan path includes the target response.

In the early-time subtraction technique³ a band-limited specular reflection is synthesized in the time-domain and subtracted from the data. This technique was reformulated in the frequency domain⁴ to improve its performance. In that modification the frequency-domain GPR measurements are represented by a parametric model consisting of a sum of damped exponential terms. The unknown parameters are estimated using a total-least-squares (TLS) Prony technique.

Van der Merwe⁵ improved the TLS-Prony method⁴ by using an iterative technique. That method involves frequency-domain data collected at each antenna position. It removes spectral components different from a reference (target) signature. Clutter components distinct from that signature are successfully removed both in early time and late time. The need for a priori knowledge of the target signature is not unrealistic, since there usually exists some information regarding the mine population in a given region. For practice, however, a number of reference signatures must be examined to ensure that a detection is not missed. Further, Van der Merwe has shown that even when the reference signature corresponding to the measured mine is utilized, the algorithm can inadvertently remove it if the measured signature is sufficiently distorted due to tilting of the mine, interactions with the ground interface, or differences in the ground properties.

Brunzell⁶ proposed a method in which the time delay of each GPR scan was estimated by using nonlinear optimization. By stacking a sequence of system impulse responses with these estimated delays a ground reflection was synthesized and subsequently subtracted from the measured data. This method works well, even with GPR data taken over soil having an uneven profile, provided the magnitude of the ground reflection is uniform across the GPR map. Because of factors such as variations in soil moisture content, variations in the magnitude of the ground reflection are often encountered in data collected over real ground and, hence, Brunzell's method is often unsatisfactory in practice.

Limitations in the existing techniques led us to develop the method proposed here. Our work is an extension of the method of Brunzell, in which spatial variations in both the ground reflection magnitude and delay are simultaneously estimated using nonlinear optimization. Because these variations are relatively smooth functions of position, we model them using low-order polynomials. These polynomials cannot track the rapid spatial variations that occur when a buried mine is encountered and, as a result, the ground reflection estimates are only weakly affected by the presence of targets.

To illustrate the process, consider the response of a one-inch deep TS-50 mine shown in Figure 1(a). This figure shows a plot of signal amplitude as functions of time/depth (vertically) and antenna position (horizontally). The reflection from the air-ground interface, which occurs around -0.5 ns, dominates the data, while the hyperbolic arc produced by the mine is barely visible. The smooth spatial variations in the ground reflection magnitude and the time delay observed in Figure 1(a) are typical of GPR data collected in real ground.

To proceed, split the measured data into N_{seg} spatial segments as indicated in Figure 1(b). Over each segment we approximate the spatial variation in both the amplitude and time delay of the ground reflection by weighted sums of low-order polynomials. The quality of the model fit improves as the segment length is reduced, but when the segment length is comparable to the dimension of a mine, then the presence of a mine improperly biases the model. To avoid this effect the segment length should be restricted to dimensions larger than a mine. To minimize redundancy in the estimate it is preferable to use orthogonal polynomials. In this work the domain of the segment was normalized to the interval $[-1,1]$, and Chebyshev polynomials were used.

We formulate the approximate reflection as follows: Let $A_i(x)$ denote the spatially varying ground reflection amplitude and $B_i(x)$ the time delay of the ground reflection peak over segment i . We approximate $A_i(x)$ and $B_i(x)$

Figure 1. Raw GPR map generated from data collected over a one inch deep TS-50 mine.

as the sums

$$A_i(x) = \sum_{n=0}^4 a_{in} T_n(x) \quad (1)$$

$$B_i(x) = \sum_{n=0}^4 b_{in} T_n(x) \quad (2)$$

where $T_n(x)$ are the Chebyshev polynomials defined by the recursive relation

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x), \quad n \geq 1 \quad (3)$$

with $T_0(x) = 1$ and $T_1(x) = x$. Because the reflection coefficient at the air-ground interface is typically complex, the coefficients a_{in} in (1) are represented by real and imaginary parts. The coefficients

$$\mathbf{a}_i = [a_{i0}^{real}, a_{i0}^{imag}, a_{i1}^{real}, a_{i1}^{imag}, \dots] \quad (4)$$

$$\mathbf{b}_i = [b_{i0}, b_{i1}, \dots] \quad (5)$$

are unknown parameters to be estimated.

We estimate the frequency-domain ground reflection in spatial segment i as

$$G_i^{est}(x, f) = A_i(x)H(f) \exp^{-j2\pi f B_i(x)} \quad (6)$$

where $H(f)$ is the windowed frequency response of the radar, i.e.,

$$H(f) = \bar{H}(f)W(f) \quad (7)$$

in which $\bar{H}(f)$ is the average frequency spectrum of the raw GPR data within the segment and $W(f)$ is a window applied when creating the time-domain data. In our work a Hanning window was used. The time-domain reflection estimate g_i^{est} becomes

$$g_i^{est}(x, t, \mathbf{a}_i, \mathbf{b}_i) = A_i(x)h(t - B_i(x)) \quad (8)$$

where $h(t)$ is the inverse Fourier transform of $H(f)$. The measured ground reflection over segment i can then be expressed as

$$g_i^{meas}(x, t) = g_i^{est}(x, t, \mathbf{a}_i, \mathbf{b}_i) + n_i(x, t) \quad (9)$$

Figure 2. Linear weighting functions used for blending $A_i(x)$ and $B_i(x)$ functions.

where $n_i(x, t)$ represents the modeling error. The coefficients $\mathbf{a}_i$ and $\mathbf{b}_i$ are found by a least-square error process, which requires a nonlinear optimization of

$$[\hat{\mathbf{a}}_i, \hat{\mathbf{b}}_i] = \arg \min_{\mathbf{a}_i, \mathbf{b}_i} \|g_i^{meas}(x, t) - g_i^{est}(x, t, \mathbf{a}_i, \mathbf{b}_i)\|^2 \quad (10)$$

Once the weighting coefficients $\mathbf{a}_i$ and $\mathbf{b}_i$ are estimated, the amplitude term $A_i(x)$ and the delay term $B_i(x)$ of segment i are evaluated using (1).

To synthesize the ground reflection over the entire antenna scan, it is necessary to merge estimates obtained within each segment. Discontinuities at the edges of these overlapping segments are avoided by blending the $A_i(x)$ and $B_i(x)$ functions using the set of linear weighting functions shown in Figure 2.

Parameter estimation was performed using a non-linear least square error minimization function, *fminu*, in the Matlab Optimization Toolbox. We observed that the optimization routine would not converge for several data sets. Performance was improved by using a recursive approach. The parameters a_{i0}^{real} , a_{i0}^{imag} and b_{i0} were estimated for a zero-order approximation. These estimates were then used as the initial guesses for a_{i0}^{real} , a_{i0}^{imag} and b_{i0} in a first-order approximation, with zero initial values for the remaining parameters, a_{i1}^{real} , a_{i1}^{imag} and b_{i1} . This procedure was repeated until the parameters at the desired order were estimated.

Because our goal is accurate estimation of the ground bounce term, the above procedure was applied only to the depth interval that contained the ground bounce. To identify that interval, the ensemble sum of energy in all range profiles was computed by summing the signature energy in the along-scan direction. Data lying within time indices that encompassed 99% of the total energy were used for the estimation process.

3. REMOVING STATIONARY CLUTTER VIA SUBSPACE DECOMPOSITION

The above-described polynomial model eliminates most of the ground reflected clutter, which dominates GPR data acquired by down-looking antennas. This algorithm, however, does not eliminate other clutter due to antenna ringing, cable mismatch, etc., which typically appear at later times in the form of horizontal bands. Although such clutter can be eliminated, in principle, by proper system design and calibration, residual amounts are invariably present in any real system. These clutter contributions are small compared to the ground reflection, but they are undesirable because they may mask weak returns from deeper mines.

To remove these clutter bands, which are nearly spatially independent, we adopt a subspace decomposition technique proposed by Marinovich⁷ for detecting non-stationary signals in stationary noise. For completeness, we summarize that formulation below.

Let W_N ($N \times M$), $M \leq N$ be a stationary noise (clutter) matrix generated by replicating M copies of a column vector comprising a GPR impulse response waveform. A signal matrix W_S ($N \times M$) contains the GPR response acquired when an antenna scans over a buried point scatterer. W_S has the familiar hyperbolic arc-shaped response.

Because the noise is assumed spatially stationary (i.e., it has equal columns), W_N has unit rank. Conversely, W_S is non-stationary and has rank greater than unity.

Now consider the generalized singular value decomposition (GSVD) of the signal W_S with respect to the noise W_N . In general, the GSVD yields a matrix representation of the form

$$W_S = U_S D_S X^{-1} \quad (11)$$

$$W_N = U_N D_N X^{-1} \quad (12)$$

Matrices D_S and D_N are diagonal with the elements

$$D_S = \text{diag}(\alpha_i), \quad 0 \leq \alpha_1 \leq \alpha_2 \leq \dots \leq \alpha_M \quad (13)$$

$$D_N = \text{diag}(\beta_i), \quad \beta_1 \geq \dots \geq \beta_q \geq \beta_{q+1} = \dots = \beta_M = 0 \quad (14)$$

where $q = \text{rank}(W_N)$. Columns of X , denoted by $x^{(i)}$, are the generalized singular vectors of W_N and W_S satisfying

$$\beta_i^2 W_S^T W_S x^{(i)} = \alpha_i^2 W_N^T W_N x^{(i)} \quad (15)$$

Because $\text{rank}(W_N) = q = 1$ for stationary noise, only $\beta_1 \neq 0$. Now isolate the noise subspace in X , U_S , U_N and D_S as follows:

$$X = \begin{bmatrix} x^{(1)} & X_1 \end{bmatrix}, \quad U_S = \begin{bmatrix} u_S^{(1)} & U_{S1} \end{bmatrix}, \quad U_N = \begin{bmatrix} u_N^{(1)} & U_{N1} \end{bmatrix} \quad (16)$$

$$D_S = \begin{bmatrix} \alpha_1 & 0 \\ 0 & D_1 \end{bmatrix}, \quad D_N = \begin{bmatrix} \beta_1 & 0 \\ 0 & 0 \end{bmatrix} \quad (17)$$

Post-multiplying Eqns. (11) and (12) by X and using the relations in (16) and (17) yields

$$W_S X = U_S D_S = \begin{bmatrix} \alpha_1 u_S^{(1)} & U_{S1} D_1 \end{bmatrix} \quad (18)$$

$$W_N X = U_N D_N = \begin{bmatrix} \beta_1 u_N^{(1)} & O \end{bmatrix} \quad (19)$$

which can be expanded to produce

$$W_S x^{(1)} = \alpha_1 u_S^{(1)} \quad (20)$$

$$W_S X_1 = U_{S1} D_1 \quad (21)$$

$$W_N x^{(1)} = \beta_1 u_N^{(1)} \quad (22)$$

$$W_N X_1 = O \quad (23)$$

Eqns. (20)-(23) imply that all stationary noise energy and a small fraction of signal energy lie in a space defined by the first generalized singular vector $x^{(1)}$. Because α_1 is small, W_S is mapped in (21) with minimum loss of signal power. There is no noise energy in the subspace defined by the remaining generalized singular vectors (X_1). Because X^{-1} can be ill-conditioned, Marinovich suggests using a QR-decomposition

$$(X^{-1})^T = QR \quad (24)$$

with which (11) and (12) can be put in the form

$$W_S Q = U_S D_S R^T \quad \text{and} \quad (25)$$

$$W_N Q = U_N D_N R^T \quad (26)$$

If Q is expressed as $Q = [q^{(1)} \quad Q_1]$, then a projection matrix $P = Q_1 Q_1^T$ can be defined such that

$$W_S P \approx W_S \quad \text{and} \quad (27)$$

$$W_N P = 0 \quad (28)$$

For each clutter-reduced data set we evaluate the above projection matrix P and apply it to that data. This projection is a useful method for enhancing the detection of small anti-personnel mines, which usually have weak, localized signatures that are easily masked by clutter. The projection reduced stationary clutter, but (as shown below) did not significantly affect the signatures of small mines. Mine signatures with relatively slow spatial variation (for example, signatures of large anti-tank mines such as the TM-46) are adversely affected by this clutter removal step. Because anti-tank mines typically have strong returns, they can often be detected easily and do not require the use of this processing.

4. SUBSURFACE IMAGING

Imaging techniques can be used to focus the energy present in a point target's hyperbolic arc back to a single point. Synthetic aperture radar is based on a closely related concept. The seismic community has developed a technique known as migration for performing this processing. Migration is well suited to GPR processing and is explored here.

Several techniques have been developed for migration. We will employ an approach based on the scalar wave equation. In this work we employ the “exploding reflector” model.⁸ Suppose that a monostatic radar collects data $d(x, t)$, where x is the scan dimension. Since the total signal propagation path is twice the one-way path, the signal reflected back to a monostatic radiator is the same as that received by a passive sensor when (1) the reflector becomes a radiator and (2) the velocity of propagation is reduced by a factor of two. The concept simplifies the analysis that follows.

We will employ frequency-wavenumber (ω - k) migration, which is an efficient spectral technique. The derivation of this standard technique appears in several works^{9–11} and only the final result appears below. The time-dependence $e^{+i\omega t}$ is implied and suppressed in what follows.

Consider a scalar 2-D field $f(x, z, t)$ produced by some source distribution in a homogeneous medium with propagation velocity v . The spectral representation of our measurements $f(x, z = 0, t)$ can be computed as

$$F(k_x, z = 0, \omega) = \int dx \int dt f(x, y = 0, z = 0, t) e^{-i\omega t + ik_x x} \quad (29)$$

Under the exploding reflector hypothesis, the field distribution of interest is $f(x, z, t = 0)$. It can be shown that this distribution is given by

$$f(x, y = 0, z, t) = \frac{1}{(2\pi)^2} \int d\omega \int dk_x e^{+i\omega t} e^{-ik_x x} F(k_x, y = 0, z = 0, \omega) e^{-iK_z z} \quad (30)$$

where

$$K_z = \sqrt{k^2 - k_x^2} \quad (31)$$

where $k = 2\omega/v$ is the wavenumber in the medium (under the exploding reflector hypothesis). We convert the integral over ω to an integral over K_z by deforming the contour of integration. We find

$$f(x, y = 0, z, t) = \frac{1}{4\pi^2} \int dK_z \int dk_x e^{+i\omega(K_z)t} e^{-ik_x(x-x')} F(k_x, y = 0, z = 0, \omega(K_z)) e^{-iK_z z} \frac{d\omega(K_z)}{dK_z} \quad (32)$$

where

$$\begin{aligned} \omega(K_z) &= \frac{v}{2} \sqrt{k_x^2 + K_z^2} \\ &= (K_z v/2) \sqrt{1 + (k_x/K_z)^2} \\ &= (K_z v/2) + (K_z v/2) (\sqrt{1 + (k_x/K_z)^2} - 1) \end{aligned} \quad (33)$$

Assuming there are no singularities in F we can deform the path of integration in K_z to the real line, which allows equation (32) to be written as a 2-D Fourier transform

$$f(x, y = 0, z, t = 0) = \int dK_z \int dk_x e^{-ik_x(x-x')} G(k_x, K_z) e^{-iK_z z} \quad (34)$$

where

$$G(k_x, K_z) = \frac{1}{4\pi^2} \frac{d\omega(K_z)}{dK_z} F(k_x, 0, 0, \omega(K_z)) \quad (35)$$

Thus, for the case of homogeneous ground, we can find the reflectivity function as a 2-D Fourier transform of the measured data, suitably interpolated.

5. EXAMPLE RESULTS

The effectiveness of our new technique is best judged from examples, which we present here.

When the polynomial-based model for surface reflection is applied to a one-inch deep TS-50 mine (cf. Figure 1(a)), it produces the estimated ground reflection shown in Figure 3. Subtracting this estimate from the raw data gives the map of Figure 4. The ground reflection term, which appeared near -0.5 ns in the raw data, is no longer visible in the clutter removed data. The most important observation to make from Figure 4 is that the signature of this very shallow mine is preserved and enhanced, something that is difficult to achieve for such shallow targets with previously described clutter removal techniques. Figure 4 contains a weak band of residual clutter between -1.5 ns and -1.0 ns. Applying the subspace-decomposition technique described above produces the result shown in Figure 5. The clutter band noted above is diminished but not totally eliminated, because this clutter is not entirely spatially invariant as assumed in the formulation.

Figure 3. The ground reflection clutter profile estimated by applying the polynomial model-based approach to measured GPR data.

Next, consider a VS-50 mine buried at a three-inch depth. We will compare the proposed approach and ensemble average subtraction. Figure 6(a) is the raw data, in which the mine signature is barely visible, and Figure 6(b) was obtained after ensemble average subtraction. Because the ground reflection profile is nonuniform, average subtraction is unsatisfactory. When the raw GPR data of Figure 6(a) are processed by applying the new clutter reduction technique, the data in Figure 6(c) are obtained. The improvement in signal-to-clutter ratio with the proposed method is clear.

Some examples of migration imaging are shown in Figures 7 and 8 for data acquired over a flush-buried VS-50 mine. The benefits of the clutter reduction process are evident upon comparing Figures 7 with the raw data and Figures 8 with the clutter-reduced data. Although migration will improve the detection of shallow targets, even when the ground clutter is present, the presence of the mine is much more evident in the clutter-reduced data.

6. CONCLUDING REMARKS

We have described a technique for suppressing ground-reflection clutter that appears in down-looking GPR systems. This technique greatly increases the signal-to-clutter ratio of near-surface targets, and it improves imaging of such mines using migration. We presented examples of both clutter reduction and imaging before and after clutter reduction.

Figure 4. Data obtained by removing the estimated ground-reflection from the measured data. This data corresponds to a one inch deep TS-50 mine.

Figure 5. Data obtained by projecting the ground-reflection removed data to reduce the spatially stationary clutter.

Figure 6. Comparison of raw and clutter-reduced data generated from measurements over a three inch deep VS-50 mine.

Figure 7. Migration of GPR data without clutter reduction.

Figure 8. Migration of GPR data with clutter reduction.

The proposed technique is a two-phase method. In the first phase the surface reflection is estimated using low-order polynomials for the reflection complex amplitude and time delay. After subtracting the reflection estimate, the residual clutter, some of which arises from defects in the radar impulse response, is suppressed using a subspace decomposition technique. The second phase should permit the GPR to detect objects at greater depth.

The proposed method has a few limitations. Although the clutter reduction technique can remove the ground reflection without affecting the signatures of small, shallow or surface targets, it will remove slowly varying signatures due to large, shallow mines. This limitation is not overly restrictive, since large anti-tank mines are usually deeply buried. In addition, a very rough surface will produce a rapidly varying surface reflection, which may be poorly modeled by the low-order polynomials employed here. Finally, we noted previously that large flat mines can have a large spatially stationary component, which can be improperly removed by the subspace decomposition technique.

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